

Possible Solution of the J/ψ Production Puzzle

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We argue that the s -channel cut contribution to J/ψ hadroproduction can be significantly larger than the usual cut contribution of the color-singlet mechanism (CSM), which is known to underestimate the experimental measurements. A scenario accounting for intermediate $c\bar{c}$ interactions is proposed that reproduces the data at low- and mid-range transverse momenta P_T from the Fermilab Tevatron and BNL Relativistic Heavy Ion Collider. The J/ψ produced in this manner are polarized predominantly longitudinally.

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Although heavy quarkonia are among the most analyzed bound quark systems, ever since the first measurements by the CDF Collaboration of the *direct* production of J/ψ and ψ' at $\sqrt{s} = 1.8$ TeV [1, 2] we are facing persistent disagreements between theoretical predictions from various models and experimental studies of the cross section and the polarization (for reviews see [3]).

The recent confirmation by CDF [4] of their previous polarization measurement [5] showing an unpolarized or slightly longitudinally polarized J/ψ yield has reinforced doubts about the applicability of the quark-velocity expansion (v) of nonrelativistic QCD (NRQCD) [6] for the rather “light” $c\bar{c}$ system. On the theory side, many new results completed — but also questioned — our knowledge of charmonium production: the long-awaited next-to-leading order (NLO) QCD corrections to the color-singlet contributions [7] are now available and show significant enhancement of the cross section; an up-to-date proof [8] of NRQCD factorization holding true at any order in v in the gluon-fragmentation channels was provided; the universality of the nonperturbative input of NRQCD was challenged by fixed-target measurements [9], similar to what had been found previously for the HERA data [3]; NRQCD factorization was shown to require modification in fragmentation regions where three heavy quarks have similar momenta [10]; finally, the c -quark fragmentation approximation was shown [11] to be only valid at much higher P_T than expected from the pioneering works of Ref. [12].

In view of the difficulty of most theoretical approaches in reproducing the experimental data, the authors of Ref. [13] undertook a systematic study of the cut contributions due to off-shell and nonstatic quarks. In particular, they questioned the assumption of the CSM that takes the heavy quarks forming the quarkonium (Q) as being on-shell [14]. If they are not, the usual s -channel cut contributes to the imaginary part of the amplitude and need to be considered on the same footing as the CSM cut. These s -channel cut contributions, which were the specific focus in Ref. [13], are illustrated in Fig. 1.

In order to provide a conserved current for such off-

shell configurations, one must introduce an additional four-point function, or contact current [15]. Dynamically such a current accounts for the interactions between the $c\bar{c}$ pair emitting the external gluon. In fact, this mechanism arises because of the possibility that the outgoing gluon stems from *within* the dressed $c\bar{c} J/\psi$ vertex, as depicted in Fig. 2. The quark pair (c_1, c_2) that makes the final J/ψ -gluon state is now in a color-octet state which thus recovers the necessity for such configurations as a natural consequence of restoring gauge invariance.

The construction of this 4-point function based on gauge invariance alone is not unique since additional transverse contributions have no bearing on current conservation. In principle, the details of this 4-point function and of the necessary transverse contributions would follow from a full dynamical treatment that consistently accounts for all such interactions. At present at least, this is beyond the scope of what is theoretically possible.

Yet, it is possible to construct a 4-point function that satisfies certain minimal requirements [15, 16]. The 4-point function proposed in [13] provided a conserved current but was not entirely satisfactory since it contained poles (similar to the basic direct and crossed contributions), and such poles for the contact current are unphysical and therefore should be avoided [15].

To this end, drawing on the experience gained in pion photoproduction processes [17, 18, 19, 20], we propose here a contact current for J/ψ production which satisfies

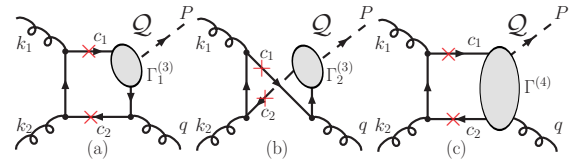


FIG. 1: (Color online). (a,b) Leading-order (LO) s -channel cut diagrams contributing to $gg \rightarrow Qg$ with direct and crossed box diagrams employing the $c\bar{c} J/\psi$ vertex. The crosses indicate that the quarks are on-shell. (c) Box diagram with $c\bar{c} Qg$ contact term mandated by gauge invariance.

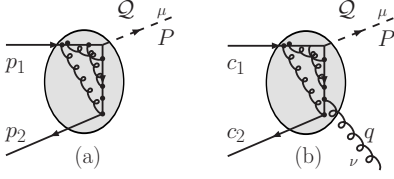


FIG. 2: Example of the mechanisms (a) contributing to the dressing of the 3-point function $\Gamma^{(3)}$ and (b) responsible for the 4-point function $\Gamma^{(4)}$: the external gluon is attached here within gluon loops of the dressed vertex, thus producing a diagram without poles and with a kinematic behavior genuinely different from the initial 3-point vertex.

the requirements of full gauge invariance, going beyond mere current conservation, in terms of the generalized Ward–Takahashi identity [16, 17]. The basic mechanism how to do this was proposed a long time ago by Drell and Lee [15] for the electromagnetic case, where it basically amounts to the minimal substitution prescription $\partial^\mu \rightarrow \partial^\mu + iQA^\mu$ (Q : charge, A^μ : vector potential) in an effective Lagrangian corresponding to the dressed hadronic vertex. As noted by Drell and Lee, this particular approach is deficient in that it violates the high-energy scaling behavior, because in avoiding poles for the 4-point function, it partially replaces the true momentum dependence of the vertices by constants. We avoid this shortcoming by building a phenomenological vertex incorporating this low-energy behavior and the expected momentum dependence of the vertices at high energies by interpolating between these two regimes. As we show below, the resulting enhanced s -channel cut contributions reproduce the experimental data from both the CDF and PHENIX collaborations up to transverse momenta $p_T \simeq 10$ GeV.

The present formalism builds on the approach developed in [13], where the transition $q\bar{q} \rightarrow Q$ is described by the 3-point function

$$\Gamma_\mu^{(3)}(p, P) = \Gamma(p, P)\gamma_\mu, \quad (1)$$

where $P \equiv p_1 - p_2$ and $p \equiv (p_1 + p_2)/2$ are the total and relative momenta, respectively, of the two quarks bound as a quarkonium state, with p_1 and p_2 being their individual four-momenta. Ansatz (1) amounts to representing the vector meson as a massive photon with a nonlocal coupling. In the present work, we describe the relative-momentum distribution $\Gamma(p, P)$ of the quarks phenomenologically as a Gaussian, as in Ref. [13], where full details can be found.

The generic picture of the physical origin of the dressed vertex function $\Gamma(p, P)$ is given in Fig. 2(a), and the ensuing contact current when coupling the gauge boson *into* this vertex is illustrated in Fig. 2(b).

The requirement of gauge invariance is usually written in terms of the generalized Ward–Takahashi relations [16] for the complete current. For the purpose of restoring

gauge invariance, it is most convenient to rewrite this into an equivalent condition for the contact current (see [17] for the analogous procedure in pion photoproduction). To this end, let us write the 4-point function depicted in Fig. 2(b) as

$$\Gamma^{(4)} = -ig_s T_{ik}^a M_c^\nu \gamma^\mu, \quad (2)$$

where g_s is the strong coupling constant, T_{ik}^a the color matrix, and μ and ν are the Lorentz indices of the outgoing J/ψ and gluon, respectively. For simplicity, we have suppressed all indices on the left-hand side. The $c\bar{c}J/\psi$ vertex function $\Gamma^{(3)}$ with the kinematics of the direct graph is denoted here by Γ_1 and for the crossed graph by Γ_2 , *i.e.*, $\Gamma_1 = \Gamma(c_1 - \frac{P}{2}, P)$ and $\Gamma_2 = \Gamma(c_2 + \frac{P}{2}, P)$, as shown in Figs. 1(a) and 1(b). The gauge-invariance condition for the contact current M_c^ν for the outgoing gluon with momentum q reads now

$$q_\nu M_c^\nu = \Gamma_1 - \Gamma_2 \quad (3)$$

since this is precisely the four-divergence contribution needed to cancel the corresponding terms arising from the four-divergences of Figs. 1(a) and 1(b).

We emphasize here that, within the present semi-phenomenological approach, the procedure to preserve QCD gauge invariance for the gluon coupling follows exactly along the lines of QED for a photon since the four-point function (2) factorizes in the color matrix and the gluon coupling. This finding is directly related to the fact that the two vertex functions on the right-hand side of the four-divergence contribution (3) arise from the gluon coupling to the two intermediate quark lines in Figs. 1(a) and 1(b) which, apart from color factors, is exactly like a photon coupling to spin-1/2 particles.

We now employ the usual construction [17, 18, 19, 20] for the contact current in terms of an auxiliary function $F = F(c_1, c_2, q)$ and put

$$M_c^\nu = \frac{(2c_2 + q)^\nu (\Gamma_1 - F)}{(c_2 + q)^2 - m^2} + \frac{(2c_1 - q)^\nu (\Gamma_2 - F)}{(c_1 - q)^2 - m^2}, \quad (4)$$

where we take $c_1^2 = c_2^2 = m^2$ and $P^2 = M^2$ from the beginning, with m and M being the masses of the quark and the J/ψ , respectively. One easily verifies that this additional contact current satisfies the gauge-invariance condition (3). It is found, in particular, that F cancels out in the four-divergence.

The function $F(c_1, c_2, q)$ must be chosen so that the current (4) satisfies crossing symmetry (*i.e.*, symmetry under the exchange $c_1 \leftrightarrow -c_2$) and is free of singularities. The latter constraint implies $F = \Gamma_0$ at either pole position, *i.e.*, when $(c_2 + q)^2 = m^2$ or $(c_1 - q)^2 = m^2$, where the constant Γ_0 is the (unphysical) value of the momentum distribution $\Gamma(p, P)$ when all three legs of the vertex are on their respective mass shells. In principle, employing gauge invariance as the only constraint, we

may take $F = \Gamma_0$ everywhere. This corresponds to the minimal substitution discussed by Drell and Lee [15] (for a complete derivation see [21]) who pointed out, however, that this does not provide the correct scaling properties at large energies, which means within the present context that $F = \Gamma_0$ would not lead to the expected P_T scaling of the amplitude. Numerically, this choice overshoots the experimental data by more than one order of magnitude at $P_T = 20$ GeV. In fact, in the large relative-momentum region, we expect the contact term and therefore the function $F(c_1, c_2, q)$ to exhibit a fall-off similar to the vertex functions themselves, contrary to the minimal substitution procedure. The simplest crossing-symmetric choice with this behavior is $F = \Gamma_1 + \Gamma_2 - \Gamma_1\Gamma_2/\Gamma_0$ [19]. The solution we propose here is to build $F(c_1, c_2, q)$ from these two limiting cases. To this end, it is natural to choose the following simple ansatz

$$F(c_1, c_2, q) = \Gamma_0 - h(c_1 c_2) \frac{(\Gamma_0 - \Gamma_1)(\Gamma_0 - \Gamma_2)}{\Gamma_0}, \quad (5)$$

where the (crossing-symmetric) function $h(c_1 c_2)$ rises to become unity for large relative momentum. Note that it is not necessary that $h(c_1 c_2)$ actually vanish at either pole since the factor on its right vanishes for $(c_2 + q)^2 = m^2$ or $(c_1 - q)^2 = m^2$ and we recover the choice of $F = \Gamma_0$ at either pole. The phenomenological choice for the interpolating function $h(c_1 c_2)$ used in our calculations is

$$h(c_1 c_2) = 1 - a \frac{\kappa^2}{\kappa^2 - (c_1 c_2 + m^2)}, \quad (6)$$

with two parameters, a and κ . We would like to emphasize at this point that this choice is in no way unique. In a manner of speaking, this choice is simply a way of parameterizing our ignorance by employing minimal properties of $\Gamma^{(4)}$. We shall not, however, discuss other choices here since our main motivation is to show that s -channel cut contributions can be large and can indeed reproduce the data — the physical picture that emerges could then be tested in other production regimes.

In the kinematical region accessed at the Tevatron, the direct J/ψ are produced by gluon fusion and a final-state gluon emission is required to conserve C -parity and provide the J/ψ with its P_T . The relevant diagrams for the LO gluon fusion process can be found in [13], the only difference in the present treatment being the new choice of $\Gamma^{(4)}$. Also, we use the same normalization of $\Gamma^{(3)}$ as in [13].

The double-differential polarized cross section in transverse momentum P_T and rapidity y is given by [13]

$$\frac{d\sigma_r}{dy dP_T} = \int_{x_{\min}}^1 dx_1 \frac{2\hat{s} P_T g(x_1) g(x_2(x_1))}{\sqrt{s}(\sqrt{s}x_1 - E_T e^y)} \frac{d\sigma_r}{dt}, \quad (7)$$

where $d\sigma_r/d\hat{t}$ is the partonic differential cross section, with $r = L, T_1, T_2$ being the quarkonium helicity, and

$\hat{s} = (k_1 + k_2)^2$, $\hat{t} = (k_2 - q)^2$ and $\hat{u} = (k_1 - q)^2$ are the Mandelstam variables for the partonic process. In the present calculations, we use the LO gluon distribution $g(x)$ of [22], and the same mass and size parameter Λ as given in Ref. [13]. In any case, our conclusion that the s -channel cut contribution can reproduce the experimental data with adjustments of the values for a and κ would not change at all over a wide range for these parameters.

Figure 3(a) shows our results with parameter values $a = 4$ and $\kappa = 4.5$ GeV for $\sqrt{s} = 1.8$ TeV in the pseudorapidity range $|\eta| < 0.6$ compared with the cross-section measurement of direct J/ψ by CDF [2], the usual LO CSM from $gg \rightarrow J/\psi g$ [14] and LO CSM from $gg \rightarrow J/\psi c\bar{c}$ [11]. Our results agree very well with the CDF data up to about $P_T = 10$ GeV. At higher P_T , our curve falls below the data as expected from the genuine $1/P_T^8$ scaling of a LO box diagram. Inclusion of higher-order corrections incorporating fragmentation-type topologies ($\sim 1/P_T^4$) and associated-production channels are expected to fill the gap between data and theory at high P_T . It is interesting to note the different P_T behaviors of σ_T and σ_L leading to a dominance of the latter at large P_T and a negative value for the polarization α . Figure 3(b) shows our results at $\sqrt{s} = 200$ GeV, still with $a = 4$ and $\kappa = 4.5$ GeV, com-

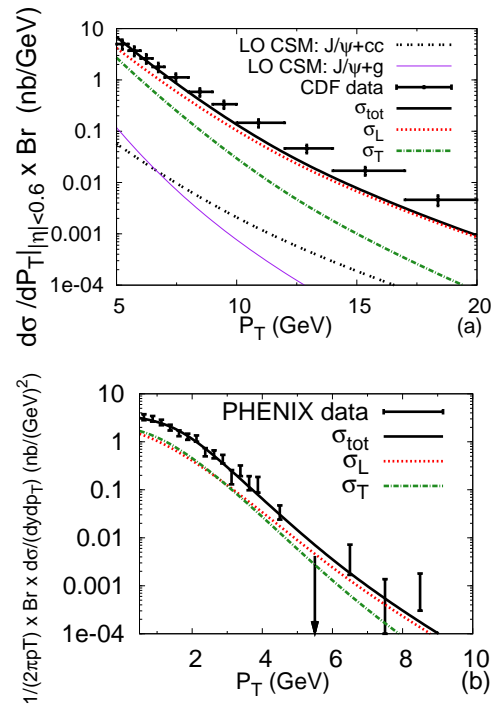


FIG. 3: (Color online). (a) Comparison between polarized (σ_T and σ_L) and unpolarized (σ_{tot}) cross sections [with parameters $a = 4$, $\kappa = 4.5$ GeV in Eq. (6)], LO CSM contributions, and CDF experimental data [2] at the Tevatron ($\sqrt{s} = 1.8$ TeV, pseudorapidity $|\eta| < 0.6$). (b) Comparison between σ_T , σ_L , σ_{tot} and PHENIX data [23] at RHIC ($\sqrt{s} = 200$ GeV, rapidity $|y| < 0.35$).

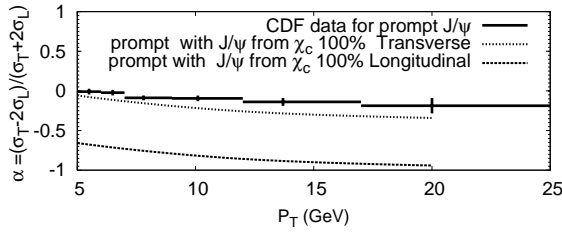


FIG. 4: Prompt J/ψ polarization: theory *vs.* CDF data [4].

pared with the PHENIX data [23].

Since J/ψ polarization measurements exist only for the prompt yield, we have computed α from our direct- J/ψ cross sections in two extreme cases, one where the J/ψ 's from χ_c are 100% transverse and another where they are 100% longitudinal, the first scenario being the more likely one. Figure 4 shows the comparison between this computation and the recent results by CDF at $\sqrt{s} = 1.96$ TeV [4].

In conclusion, we have shown here that among the two singularities in the box diagram contribution to quarkonium production at LO, the one from the s -channel cut is significantly larger than expected when one includes interactions between the quark pair which binds into the quarkonium and emits the final-state gluon. In the present work, we have accounted for the effect of those interactions by a phenomenological contact current, or 4-point function, $\Gamma^{(4)}$ which follows directly from the implementation of the full gauge-invariance requirements appropriate for a dressed vertex $\Gamma^{(3)}$. This 4-point function, however, is not fully constrained by gauge invariance which permitted us to interpolate between the minimal substitution discussed earlier by Drell and Lee and the expected large relative-momentum behavior.

As we showed, this 4-point function provides a much larger contribution than the direct and crossed ones containing 3-point functions (cf. Fig. 1), and this can easily bring about agreement with the experimental data. In NRQCD, color-octet matrix elements account for transitions between a colored heavy-quark pair into a quarkonium by soft unseen gluon emission in the final state. In the present approach, the 4-point function accounts for gluon exchanges between the heavy quarks which emit the final-state gluon. As for the matrix elements of NRQCD, which are unknown and then fit, we fixed the unconstrained parameter of this function in order to reproduce the experimental data at $\sqrt{s} = 1.8$ TeV from the CDF collaboration at the Tevatron for $P_T \lesssim 10$ GeV.

With the same parameters, we also obtain a very good description of the experimental measurements from PHENIX at RHIC at $\sqrt{s} = 200$ GeV. Moreover, our prediction for the polarization for the prompt J/ψ yield is mostly longitudinal. This looks promising since one expects contributions from the CSM cuts — which are known to be enhanced by the NLO corrections [7] — and

from the real part which is not evaluated at present time.

A similar enhancement by inclusion of the s -channel cut is expected in all production processes where the J/ψ is associated with a gluon, *e.g.*, photon-photon collision at LEP as well as in photo- and lepto-production at HERA.

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